Dispersion

We have seen that Average is a single figure which represents the entire distribution. Next step is to find how far the average represents the entire distribution. In other words we need to find how far the actual figures are situated away from the average ie we try to find the spread / dispersion / variation of actual values from the average.

Let us consider the following data:

A	B	C
100	75	180
100	150	100
100	75	20
100	100	100

Here average score is 100 runs for all 3 batsmen. It doesn't mean all of them are equally good. To know this we find deviation of observations from respective means. Greater the variation less representative the average is. Hence we have to consider the average for which such variation is the least.

The measures used for calculating dispersion are known as measures of dispersion.

Measures of Dispersion can be classified into two broad categories:

Absolute & Relative

Absolute measures give extent of variation for given set of observation, while relative measures is found to compare two or more sets of data w.r.t their variability.

Range:

The range is the simplest method of calculating dispersion. It is defined as the difference between largest and the smallest observation.

$R = L - S$

Relative measure

Coefficient of Range = $\frac{L-S}{L+S}$

Q1. Calculate the range and its coefficient

50 , 60 , 53 , 55 , 57 , 59 , 58 , 55 , 54 , 61

R = L – S = 61 – 50 = 11

Coefficient of Range = $\frac{L-S}{L+S} = \frac{61-50}{61+50} = \frac{11}{111}$ = 0.09

<u>Quartile Deviation</u>

Q.D. = $\frac{Q_3-Q_1}{2}$

Coefficient of Q.D. = $\frac{Q_3-Q_1}{Q_3+Q_1}$

Q2. Calculate Q.D and its coefficient for the following data

18 , 20, 35 , 41 , 28 , 15 , 13

Arrange the observations in ascending or descending order.

13, 15 , 18 , 20 , 28 , 35 ,41

***n* = 7 , odd**

Q1 = value of $\frac{n+1}{4}$ th observation = 2nd observation = 15

Q3 = value of $\frac{3(n+1)}{4}$ th observation = 6th observation = 35

Q.D. = $\frac{Q_3-Q_1}{2} = \frac{35-15}{2}$ = 10

Coefficient of Q.D. = $\frac{35-15}{35+15} = \frac{20}{50}$ = 0.4 = 40%

Q3. Calculate Q.D. and its coefficient for the following data

x	f	Cf
62	11	11
63	17	28
64	22	50
65	18	68
66	15	83
67	10	93
68	7	100

N = 100 , Even

Q1 = value of $\frac{n}{4}$ th observation = 25^{th} observation = 63

Q3 = value of $\frac{3n}{4}$ th observation = 75^{th} observation = 66

Q.D. = $\frac{Q_3-Q_1}{2}$ =$\frac{66-63}{2}$ = 1.5

Coefficient of Q.D. =$\frac{66-63}{66+63}$ = $\frac{3}{129}$ = 0.0232 = 2.325%

Q4. Calculate Q.D. and its coefficient for the following data

Class	f	Cf
10-15	4	4
15 – 20	10	14
20 – 25	16	30
25 – 30	22	52
30 – 35	10	62
35 – 40	8	70
40 - 45	6	76
45 - 50	4	80

N = 80 , Even

Q1 Class is the class containing $\frac{N}{4}$ th $= \frac{80}{4}$th $= 20^{th}$ observation ie 20-25

$$Q1 = l_1 + \frac{\left(\frac{N}{4} - cf\right) * (l_2 - l_1)}{f}$$

$$= 20 + \frac{\left(\frac{80}{4} - 14\right) * (25 - 20)}{16}$$

$$= 20 + \frac{(20 - 14) * (25 - 20)}{16}$$

$$= 20 + \frac{(6) * (5)}{16} = 20 + 1.875 = 21.875$$

Q3 Class is the class containing $\frac{3N}{4}$ th $= \frac{3*80}{4}$th $= 60^{th}$ observation ie 30-35

$$Q3 = l_1 + \frac{\left(\frac{3N}{4} - cf\right) * (l_2 - l_1)}{f}$$

$$= 30 + \frac{\left(\frac{3*80}{4} - 52\right) * (35 - 30)}{10}$$

$$= 30 + \frac{(60 - 52) * (35 - 30)}{10} = 30 + \frac{(8) * (5)}{10} = 34$$

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{34 - 21.875}{2} = 6.0625$$

$$\text{Coefficient of Q.D.} = \frac{34 - 21.875}{34 + 21.875} = \frac{12.125}{55.875} = 0.217 = 21.7\%$$

Q5. Calculate Q.D. and its coefficient for the following data

Weight	No. of persons
70-79	12
80 – 89	18
90 – 99	35
100 – 109	42
110 – 119	50
120 –129	45
130 -139	20
140 - 149	10

Inclusive intervals should be changed to exclusive intervals

Weight	No. of persons	Cf
70.5-79.5	12	12
79.5– 89.5	18	30
89.5– 99.5	35	65
99.5– 109.5	42	107
109.5– 119.5	50	157
119.5–129.5	45	202
129.5-139.5	20	222
139.5– 149.5	10	232

N = 232 , Even

Q1 Class is the class containing $\frac{N}{4}$th $= \frac{232}{4}$th $= 58^{th}$ observation ie 89.5-99.5

$$Q1 = l_1 + \frac{\left(\frac{N}{4} - cf\right) * (l_2 - l_1)}{f}$$

$$= 89.5 + \frac{\left(\frac{232}{4} - 30\right) * (99.5 - 89.5)}{35}$$

$$= 89.5 + \frac{(58 - 30) * (99.5 - 89.5)}{35}$$

$$= 89.5 + \frac{28 * (10)}{35} = 89.5 + 8 = 97.5$$

Q3 Class is the class containing $\frac{3N}{4}$ th $= \frac{3*232}{4}$th $= 174^{th}$ observation

ie 119.5-129.5

$$Q3 = l_1 + \frac{\left(\frac{3N}{4} - cf\right)*(l_2 - l_1)}{f}$$

$$= 119.5 + \frac{\left(\frac{3*232}{4} - 157\right)*(129.5 - 119.5)}{45}$$

$$= 119.5 + \frac{(174 - 157)*(129.5 - 119.5)}{45} = 119.5 + 3.77 = 123.27$$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{123.27 - 97.5}{2} = 12.885$$

$$\text{Coefficient of Q.D.} = \frac{123.27 - 97.5}{123.27 + 97.5} = \frac{25.77}{220.77} = 0.1167 = 11.67\%$$

Q6. Calculate Q.D. and its coefficient for the following data

Units consumed	No. of consumers	cf
0-200	18	18
200 –400	27	45
400 – 600	32	77
600 – 800	47	124
800 – 1000	36	160
1000 –1200	40	200

N = 200 , Even

Q1 Class is the class containing $\frac{N}{4}$ th $= \frac{200}{4}$th $= 50^{th}$ observation ie 400 - 600

$$Q1 = l_1 + \frac{\left(\frac{N}{4} - cf\right)*(l_2 - l_1)}{f}$$

$$= 400 + \frac{\left(\frac{200}{4} - 45\right)*(600 - 400)}{32}$$

$$= 400 + \frac{(50 - 45)*(600 - 400)}{32}$$

$= 400 + \frac{(5)*(200)}{32} = 400 + 31.25 = 431.25$

Q3 Class is the class containing $\frac{3N}{4}$ th $= \frac{3*200}{4}$th $= 150^{th}$ observation ie 800-1000

$Q3 = l_1 + \frac{\left(\frac{3N}{4} - cf\right)*(l_2 - l_1)}{f}$

$= 800 + \frac{\left(\frac{3*200}{4} - 124\right)*(1000-800)}{36}$

$= 800 + \frac{(150-124)*(1000-800)}{36}$

$= 800 + \frac{(26)*(200)}{36} = 800 + 144.44 = 944.44$

$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{944.44-431.25}{2} = \frac{513.19}{2} = 256.595$

Coefficient of Q.D. $= \frac{944.44-431.25}{944.44+431.25} = \frac{513.19}{1375.69} = 0.373 = 37.3\%$

Mean Deviation

Mean Deviation from Mean

Deviation means dispersion or difference.

Deviation of observations from mean is $x - \bar{x}$

Absolute Deviation is $|x - \bar{x}|$

Mean Absolute Deviation is $\frac{\sum |x - \bar{x}|}{n}$ **(without frequency)**

Mean Absolute Deviation is $\frac{\sum f\ |x - \bar{x}|}{\sum f}$ **(with frequency)**

Relative measure – Coefficient of Mean Absolute Deviation is

$$\frac{\text{Mean Absolute Deviation from Mean}}{\text{Mean}}$$

Mean Deviation from Median

Mean Absolute Deviation is $\frac{\sum |x - M|}{n}$ **(without frequency)**

Mean Absolute Deviation is $\frac{\sum f\,|x-M|}{\sum f}$ **(with frequency)**

Relative measure – Coefficient of Mean Absolute Deviation is

$$\frac{\textbf{Mean Absolute Deviation from Median}}{\textbf{Median}}$$

Mean Deviation from Mode

Mean Absolute Deviation is $\frac{\sum |x-Z|}{n}$ **(without frequency)**

Mean Absolute Deviation is $\frac{\sum f\,|x-Z|}{\sum f}$ **(with frequency)**

Relative measure – Coefficient of Mean Absolute Deviation is

$$\frac{\textbf{Mean Absolute Deviation from Mode}}{\textbf{Mode}}$$

Mean Deviation from A

Mean Absolute Deviation is $\frac{\sum |x-A|}{n}$ **(without frequency)**

Mean Absolute Deviation is $\frac{\sum f\,|x-A|}{\sum f}$ **(with frequency)**

Relative measure – Coefficient of Mean Absolute Deviation is

$$\frac{\textbf{Mean Absolute Deviation from A}}{\textbf{A}}$$

Q1. Calculate Mean Deviation from Mean and its coefficient

45 , 65 , 66 , 72 , 80 , 90, 93 , 85 , 88 , 96

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
45	-33	33
65	-13	13
66	-12	12
72	-6	6
80	2	2
90	12	12
93	15	15
85	7	7
88	10	10
96	18	18
Total		128

$$\bar{x} = \frac{\sum x}{n} = \frac{780}{10} = 78$$

$$\text{Mean Deviation from Mean} = \frac{\sum |x - \bar{x}|}{n} = \frac{128}{10} = 12.8$$

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Absolute Deviation from Mean}}{\text{Mean}} = \frac{12.8}{78}$$

$$= 0.1641 \quad = 16.41\ \%$$

Q2. Calculate Mean Deviation from Median

45 , 65 , 66 , 72 , 80 , 90, 93 , 85 , 88 , 96

Arranged observations

45 , 65 , 66 , 72 , 80 , 85 , 88 , 90 , 93 , 96

x	$x - M$	$\|x - M\|$
45	-37.5	37.5
65	-17.5	17.5
66	-16.5	16.5
72	-10.5	10.5
80	-2.5	2.5
85	2.5	2.5
88	5.5	5.5
90	7.5	7.5
93	10.5	10.5
96	13.5	13.5
Total		124

N = 10 , even

$$\text{Median} = \frac{\text{Value of 5th observation + Value of 6th observation}}{2}$$

$$= \frac{80+85}{2} = 82.5$$

$$\text{Mean Deviation from Median} = \frac{\sum |x - M|}{n} = \frac{124}{10} = 12.4$$

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Absolute Deviation from Median}}{\text{Median}}$$

$$= \frac{12.4}{82.5} = 0.1503 = 15.03\%$$

Q3. Calculate Mean Deviation from Mean and its coefficient

x	f	fx	$\lvert x - \bar{x} \rvert$	f $\lvert x - \bar{x} \rvert$
5	3	15	10	30
10	4	40	5	20
15	6	90	0	0
20	5	100	5	25
25	1	25	10	10
30	1	30	15	15
Total	20			100

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{300}{20} = 15$$

Mean Deviation from Mean $= \frac{\sum f \lvert x - \bar{x} \rvert}{\sum f} = \frac{100}{20} = 5$

Coefficient of Mean Deviation $= \frac{\text{Mean Absolute Deviation from Mean}}{\text{Mean}} = \frac{5}{15} = 0.3333$

$=33.33\%$

Q4. Calculate Mean Deviation from Mean and its coefficient

Age	No of Employees	x	fx	$\|x - \bar{x}\|$	f $\|x - \bar{x}\|$
20-22	70	21	1470	6.08	425.6
22-24	90	23	2070	4.08	367.2
24-26	110	25	2750	2.08	228.8
26-28	140	27	3780	0.08	11.2
28-30	130	29	3770	1.92	249.6
30-32	80	31	2480	3.92	313.6
32-34	80	33	2640	5.92	473.6
Total	700		18960		2069.6

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{18960}{700} = 27.08$$

Mean Deviation from Mean = $\frac{\sum f\,|x - \bar{x}|}{\sum f} = \frac{2069.6}{700} = 2.96$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Mean}}{\text{Mean}} = \frac{2.96}{27.08} = .1093$

= 10.93%

Q5. Calculate Mean Deviation from Median and its coefficient

Length of illness	No of patients	x	cf	$\|x - M\|$	f $\|x - M\|$
0 – 14	5	7	5	24.7	123.5
14 – 28	17	21	22	10.7	181.9
28 – 42	17	35	39	3.3	56.1
42 – 56	7	49	46	17.3	121.1
56 - 70	7	63	53	31.3	219.1
Total					701.7

Median class is the class containing N/2 th = 26.5^{th} observation

Median class is 28-42

$$M = l_1 + \frac{\left(\frac{N}{2} - cf\right) * (l_2 - l_1)}{f}$$

$$= 28 + \frac{(26.5 - 22) * (42 - 28)}{35}$$

$$= 28 + \frac{(4.5) * (14)}{35} = 31.7$$

Mean Deviation from Median = $\frac{\sum f\,|x - M|}{\sum f} = \frac{701.7}{53} = 13.24$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Median}}{\text{Median}} = \frac{13.24}{31.7} =$ **.4176 = 41.76%**

Q6. Calculate Mean Deviation from Median and its coefficient

Total Sale in 00's	No of salesmen	x	cf	$\|x - M\|$	f $\|x - M\|$
0 – 100	10	50	10	200	2000
100 – 200	15	150	25	100	1500
200 – 300	50	250	75	0	0
300 – 400	20	350	95	100	2000
400 - 500	5	450	100	200	1000
Total					6500

Median class is the class containing N/2 th = 50^{th} observation

Median class is 200-300

$$M = l_1 + \frac{\left(\frac{N}{2} - cf\right) * (l_2 - l_1)}{f}$$

$$= 200 + \frac{(50 - 25) * (300 - 200)}{50}$$

$$= 200 + \frac{(25) * (100)}{50} = 200 + 50 = 250$$

Mean Deviation from Median = $\frac{\sum f\,|x - M|}{\sum f} = \frac{6500}{100} = 65$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Median}}{\text{Median}} = \frac{65}{250}$ =0.26= 26%

Q7. Calculate Mean Deviation from Mode and its coefficient

x	f	$\|x - Z\|$	f $\|x - Z\|$
4	10	3	30
5	14	2	28
6	16	1	16
7	18	0	0
8	10	1	10
9	5	2	10
Total	73		94

Mode = Most frequent observation = 7

Mean Deviation from Mode = $\frac{\sum f\,|x - Z|}{\sum f} = \frac{94}{73} = 1.28$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Mode}}{\text{Mode}} = \frac{1.28}{7}$ = 0.1839 =18.39%

Q8. Calculate Mean Deviation from Mode and its coefficient

Marks	No of students	x	$\|x - Z\|$	f $\|x - Z\|$
30 – 40	3	35	20	60
40 – 50	15	45	10	150
50 – 60	27	55	0	0
60 – 70	15	65	10	150
70 - 80	8	75	20	160
80 - 90	2	85	30	60
Total	70			580

Modal class is the class containing highest frequency = 50 -60

$$Z = l_1 + \frac{(f_1 - f_0) * (l_2 - l_1)}{2f_1 - f_0 - f_2}$$

$$= 50 + \frac{(27-15)*(60-50)}{2*27-15-15}$$

$$= 50 + \frac{(12)*(10)}{24} = 50 + 5 = 55$$

Mean Deviation from Mode = $\frac{\sum f\,|x - Z|}{\sum f} = \frac{580}{70} = 8.28$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Mode}}{\text{Mode}} = \frac{8.28}{55} = 0.1506$ **=15.06%**

Q9. Calculate Mean Deviation from 200 and its coefficient

Total Sale in 00's	No of salesmen	x	$\|x - 200\|$	f $\|x - M\|$
0 – 100	10	50	150	1500
100 – 200	15	150	50	750
200 – 300	50	250	50	2500
300 – 400	20	350	150	3000
400 - 500	5	450	250	1250
Total				9000

Mean Deviation from 200 = $\frac{\sum f\,|x - 200|}{\sum f} = \frac{9000}{100} = 90$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Median}}{\text{Median}} = \frac{90}{200}$ **=0.45= 45%**

Q10. Calculate Mean Deviation from Mean ,Median , Mode and their coefficients

Rainfall	No of days
0 – 10	15
10 – 20	20
20 – 30	40
30 – 40	20
40 - 50	5
Total	

Mean Deviation from Mean

Rainfall	No of days	x	fx	$\|x - \bar{x}\|$	f $\|x - \bar{x}\|$
0 – 10	15	5	75	18	270
10 – 20	20	15	300	8	160
20 – 30	40	25	1000	2	80
30 – 40	20	35	700	12	240
40 - 50	5	45	225	22	110
Total	100		2300		860

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{2300}{100} = 23$$

Mean Deviation from Mean = $\frac{\sum f\,|x - \bar{x}|}{\sum f} = \frac{860}{100} = 8.6$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Mean}}{\text{Mean}} = \frac{8.6}{23} = .3739 =$ 37.39%

Mean Deviation from Median

Rainfall	No of days	x	cf	$\|x - M\|$	f $\|x - M\|$
0 – 10	15	5	15	18.75	281.25
10 – 20	20	15	35	8.75	175
20 – 30	40	25	75	1.25	50
30 – 40	20	35	95	11.25	225
40 - 50	5	45	100	21.25	106.25
Total	100				837.5

Median class is the class containing N/2 th = 50th observation

Median class is 20-30

$$M = l_1 + \frac{\left(\frac{N}{2} - cf\right) * (l_2 - l_1)}{f}$$

$$= 20 + \frac{(50-35)*(30-20)}{40}$$

$$= 20 + \frac{(15)*(10)}{40} = 20 + 3.75 = 23.75$$

Mean Deviation from Median = $\frac{\sum f\,|x - M|}{\sum f} = \frac{837.5}{100} = 8.375$

Coefficient of Mean Deviation = $\frac{\text{Mean Absolute Deviation from Median}}{\text{Median}} = \frac{8.375}{23.75}$

=0.3526= 35.26%

Mean Deviation from Mode

Rainfall	No of days	x	$\|x - Z\|$	f $\|x - Z\|$
0 – 10	15	5	20	300
10 – 20	20	15	10	200
20 – 30	40	25	0	0
30 – 40	20	35	10	200
40 - 50	5	45	20	100
Total	100			800

Modal class is the class containing highest frequency = 20 -30

$$Z = l_1 + \frac{(f_1 - f_0) * (l_2 - l_1)}{2f_1 - f_0 - f_2}$$

$$= 20 + \frac{(40-20) * (30-20)}{2*40-20-20}$$

$$= 20 + \frac{(20) * (10)}{40} = 20 + 5 = 25$$

Mean Deviation from Mode $= \frac{\sum f\,|x - Z|}{\sum f} = \frac{800}{100} = 8$

Coefficient of Mean Deviation $= \frac{\text{Mean Absolute Deviation from Mode}}{\text{Mode}} = \frac{8}{25} = 0.32$

=32%